

# **INVENTORY MODELS AND INVENTORY EFFECTS<sup>\*</sup>**

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# INVENTORY MODELS AND INVENTORY EFFECTS

## Abstract

Traditional economic models of price-setting focus on call-auction markets in which all trading occurs simultaneously, at pre-established discrete times, with no market makers involved. Such models leave no role for any of the three sources of friction found in modern models of market liquidity: inventory, order-processing costs, and adverse selection. As market microstructure research has developed, researchers studying the links between market-maker inventories and liquidity have shed considerable light on how market makers (often modeled as dealers) resolve temporal imbalances in the continuous trading environment that characterizes most financial markets. In general, inventory models predict that market makers set ask prices above bid prices, that they lower their quotes when they have very large inventory positions, and that they may or may not change the magnitude of their quoted spreads as their inventory changes, depending on whether they are capital constrained or not. Models in which market makers face capital constraints also offer an explanation of flight to quality, in which the riskiest securities suffer the greatest liquidity declines. Multi-dealer inventory models predict that relative inventory positions give rise to interdealer trading and determine which dealers have the best (lowest ask or highest bid) quotes in a market. These predictions have been tested and largely borne out in empirical studies to date.

## INTRODUCTION

Traditional economic models of price-setting focus on call-auction markets in which all trading occurs simultaneously, at pre-established discrete times, with no market makers involved. Such models leave no role for any of the three sources of friction found in modern models of market liquidity: inventory, [order-processing costs](#), and [adverse selection](#). As market microstructure research has developed, researchers studying the links between inventories and liquidity have shed considerable light on how market makers resolve temporal imbalances in the continuous trading environment that characterizes most financial markets.

## LINKING INVENTORIES AND LIQUIDITY

The study of how inventories affect pricing and liquidity in financial markets began with the work of Garman [6]. Garman models continuous trading with a market maker, or dealer, who smoothes out temporary imbalances in other traders' orders to buy and sell securities. In Garman's model there is a single risk-neutral, monopolistic dealer who faces [stochastic](#) order arrival. The dealer has no ability to borrow either cash or securities, and he sets prices at which he is willing to sell (ask price) and buy (bid price) only once, before trading begins. The dealer's objective is to maximize his expected profit per unit of time subject to avoiding bankruptcy or failure. This formulation gives rise to a classic gambler's ruin problem. By setting his ask price higher than his bid price (a positive [bid-ask spread](#)), the dealer protects himself from certain failure, although he still faces a positive probability of failure. A key contribution of Garman's model is the idea that a

dealer's inventory affects his viability. A limitation of his model is that all prices are set before trading begins; the dealer cannot adjust his prices as his inventory changes.

## **INVENTORY MODELS WITH NO POSSIBILITY OF DEALER FAILURE**

Amihud and Mendelson [1] build on Garman's intuition to study how a dealer changes his prices as his inventory changes, explicitly incorporating inventory into the dealer's pricing problem. As in Garman's model, there is a single risk-neutral, monopolistic dealer whose objective is to maximize expected profit, but Amihud and Mendelson assume that inventory is bounded above and below by exogenous parameters, eliminating the possibility of dealer failure. This allows a tighter focus on the dealer's optimization problem. This model incorporates a semi-[Markov process](#) in which inventory is the state variable. The decision variables (bid and ask prices) depend on the level of the state variable (inventory) – thus the bid and ask prices change over time as the level of inventory changes. Amihud and Mendelson's model predicts that the dealer's optimal ask price is above his bid price, but in this model the positive spread reflects the dealer's market power. (Extending the model to include competitive dealers would drive the spread to zero.) In addition, Amihud and Mendelson find that the dealer has a preferred or target inventory position, and he adjusts his prices to return to his target inventory. If the dealer is too long, to reduce inventory towards his target level he lowers both the bid and ask prices to induce other traders to buy. The reverse is true if the dealer is below his target inventory: To raise inventory towards its target level the market maker raises both the bid and ask prices to induce other traders to sell. Amihud and Mendelson's model thus predicts that the level of bid and ask prices is a monotonically decreasing

function of the dealer's inventory. In the special case of linear supply and demand, higher inventory leads to wider spreads.

The models of Stoll [19] and Ho and Stoll [10, 11] maintain the assumption that the dealer cannot go bankrupt, but they focus on how a risk-averse dealer's inventory, [order-processing costs](#), and [adverse selection](#) risk affect his pricing. The dealer is willing to alter his portfolio away from his desired position in order to accommodate the trading desires of others. The dealer is risk averse and cannot hedge his inventory exposure, so he must be compensated for bearing the risk of changing his portfolio from his target level. In these models the spread between bid and ask prices arises as compensation for the portfolio risk borne by a risk-averse dealer. Inventory causes the dealer to move both bid and ask prices up (if he is below his target portfolio) or down (if he is above his target portfolio) by the same amount, so inventory affects the level of bid and ask prices but not the magnitude of the spread between them. But here the spread reflects dealer risk aversion, rather than market power as in Amihud and Mendelson [1]. Ho and Stoll [10, 11] demonstrate that the intuition of inventory affecting a dealer's quote levels but not the magnitude of his spread is robust to multi-period and multi-dealer configurations. In a multi-dealer setting, Ho and Stoll [11] further predict that relative inventory positions among dealers determine the amount of interdealer trading.

## **INVENTORY MODELS WITH CAPITAL CONSTRAINTS**

Most of the classical inventory models [1, 10, 11, 16, 19] assume either explicitly or implicitly that dealers have access to unlimited additional capital. Recent work by Brunnermeier and Pedersen [2] and Gromb and Vayanos [7] relaxes this assumption,

examining how liquidity provision is affected by the inventory of dealers or arbitrageurs who face capital constraints.

Brunnermeier and Pedersen [2] construct a model linking an asset's market liquidity with market makers' funding, explicitly acknowledging that market makers generally cannot raise new capital instantly. In this model market makers are risk-neutral and their objective is to maximize final wealth, subject to the constraint that the total margin (the amount of capital required to finance their inventory positions) cannot exceed their capital. Brunnermeier and Pedersen show that when market makers' margin requirements approach their available capital, market makers provide less liquidity to the market and bid-ask spreads widen. This model thus predicts that very large inventory positions lead to wider spreads, in contrast to the capital-unconstrained models in which inventories have no effect on the magnitude of bid-ask spreads. Further, Brunnermeier and Pedersen predict that when large inventory positions push market makers close to their capital limits, market makers provide liquidity mostly in lower-risk securities, causing a "flight to quality" in which risky securities become especially illiquid.

Similar predictions can be obtained from the Gromb and Vayanos [7] capital-constraints model, although their main predictions concern the welfare implications of capital-constrained market makers.

## **EMPIRICAL EVIDENCE ON INVENTORY EFFECTS**

Models of how market maker inventories affect pricing and liquidity in financial markets all predict that a market maker will set his ask price above his bid price for reasons including bankruptcy risk (Garman [6]), market power (Amihud & Mendelson

[1]), and risk aversion (Stoll [19], Ho & Stoll [10, 11]). This prediction of a positive bid-ask spread is borne out by observations in virtually all financial markets.

All of the inventory models also predict that inventories affect the level of bid and ask prices, with market makers lowering prices to reduce their inventory when they hold large positions and raising prices when their inventory positions are low or negative.

Lyons [12] and Cao, Evans, and Lyons [3] find evidence of such inventory-induced quote shading in the foreign exchange market, as do Garleanu, Pedersen, and Poteshman [5] in the S&P500 index options market. However, in the futures market, Manaster and Mann [15] find that futures market makers (who differ somewhat from the typical dealer in inventory models) actually raise their prices when they have larger inventory positions, in contradiction of the inventory models' prediction. Using London Stock Exchange (LSE) data, Hansch, Naik, and Viswanathan [8], Reiss and Werner [18], and Naik and Yadav [17] find support for market makers' controlling risk by mean-reverting their inventory positions towards a target level. Hasbrouck and Sofianos [9] and Madhavan and Sofianos [14] find evidence that specialists on the New York Stock Exchange (NYSE) have preferred inventory positions and that their inventories affect prices, but Madhavan and Smidt [13] also find that specialist inventories deviate from their target levels for periods as long as several weeks.

Several researchers analyze the link between inventories and liquidity suggested by inventory models. Consistent with the predictions of Ho and Stoll [11] for a market with competitive dealers, Hansch, Naik, and Viswanathan [8] and Reiss and Werner [18] find that differences in inventories across dealers on the LSE affect the extent of interdealer trading, but that individual dealer inventories do not affect the magnitude of each dealer's

quoted spread intraday. On an interday level, Comerton-Forde, Hendershott, Jones, Moulton, and Seasholes [4] find that large NYSE specialist inventories lead to wider spreads, consistent with the predictions of capital-constraints models such as Brunnermeier and Pedersen [2]. Comerton-Forde et al. [4] also find evidence that when NYSE specialist inventories are particularly high, spreads widen more for high-risk than for low-risk stocks, supporting Brunnermeier and Pedersen's [2] inventory explanation for flight to quality.

## **SUMMARY**

Dealer inventory was among the first market frictions studied by market microstructure theorists. Inventory models predict that dealers set ask prices above bid prices, that they lower their quotes when they have very large inventory positions, and that they may or may not change the magnitude of their quoted spreads as their inventory changes, depending on the particular modeling assumptions. Multi-dealer inventory models further predict that relative inventory positions give rise to interdealer trading and determine which dealers have the best (lowest ask or highest bid) quotes in a market. These predictions have been tested and largely borne out in empirical studies to date, but the difficulty of obtaining data on dealer inventories has limited researchers' ability to test the models in some of the largest dealer markets, such as the fixed income market.

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